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AN AXISYMMETRIC PROBLEM FOR A LAYER WITH A SYSTEM OF THIN ELASTIC INCLUSIONS *

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An approximate method is proposed for determining the stress concentration in an infinite elastic layer near a system of thin elastic inclusions bounded by coaxial smooth surfaces of revolution. The problem is reduced to a system of integral equations of the second kind by a Hankel transformation, which can then be solved by successive approximations or numerically. A numerical computation is performed for the case of two spheroidal inclusions in unbounded space. The results are compared with those known in the literature.

1. In an elastic layer, let N thin inclusions of different geometric parameters and elastic properties, bounded by smooth surfaces of revolution V_i ($i = 1, \ldots, N$) and having plane middle surfaces parallel to the layer boundaries (Fig. 1), be contained on an axis normal to the layer boundaries. The inclusions and the layer material are connected along the whole common boundary. We introduce a cylindrical system of coordinates r, φ, z with the Oz axis coincident with the axis of revolution of the surfaces V_i. Axisymmetric forces, which cause a normal stress s_{ri} and a tangential stress f_{ri} in the middle surfaces S_i of the inclusions in the case of a homogeneous layer (S_i are circles of radii a_i) are applied to the layer boundaries. We will determine the stress concentration in the neighbourhood of the inclusions.

For an approximate solution of the problem, we replace the inclusions by cavities with a certain stress distribution on the surfaces V_i. The assumptions of small thickness of the inclusions $2h_i(r)$ ($h_i \ll a_i$), smoothness of the surfaces V_i ($|dh_i | dr \ll 1$), and stiffness of the inclusions ($E_i \leqslant E$) permit the strong interaction between the inclusions and the fundamental material to be represented approximately by the relationships /1,2/

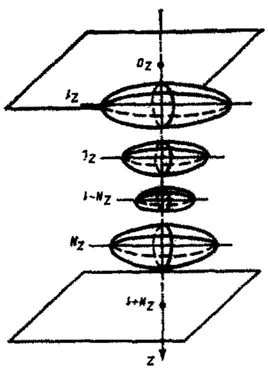

Fig.1

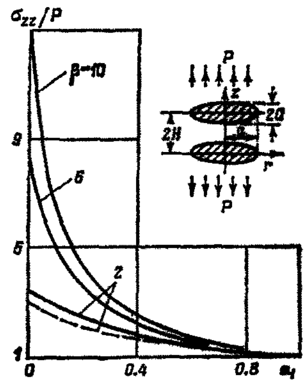

Fig.2

$$f_{zi}^*(r) = {}^1/_2\,[u_{zi}^*(r)]\,E_i/h_i(r),\quad f_{ri}(r) = {}^1/_2\,[u_{ri}^*(r)]\,\mu_i/h_i(r) \tag{1.1}$$

where $[u] = u^+ - u^-$ (u^+ and u^- are values of the quantity u on the upper and lower edges of the surfaces V_i with respect to the domains S_i, respectively), E_i, μ_i are the elastic and shear moduli of the material of the inclusions, E is the elastic modulus of the host matrix, $\mathbf{u}_i^* = \mathbf{u}_i^*(u_{zi}^*, u_{ri}^*)$ is the displacement vector of points of the surfaces V_i which is assumed to be equal approximately to the sum $\mathbf{u}_i^0 + \mathbf{u}_i$, where $\mathbf{u}_i$ is the displacement vector of points of the surface of a mathematical slit along S_i to whose edges unknown stresses $\mathbf{f}_i^* - \mathbf{f}_i$, $\mathbf{f}_i = \mathbf{f}_i(f_{zi}, f_{ri})$ are applied, and $\mathbf{u}_i^0$ is the known displacement vector of points of the surface v_i in the case of a homogeneous layer subjected to given forces on the boundary.

It is known /3,4/ that the state of stress in the neighbourhood of the tip portion of a thin cavity can be represented in terms of stress intensity factors for an equivalently stressed mathematical slit along the middle surface of the cavity.

Therefore, the problem of inhomogeneities reduces to a certain boundary value problem in elasticity theory for a layer with the slits S_i

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$$\sigma_{zz}(r, z_0) = \sigma_{zz}(r, z_{N+1}) = 0 \qquad (0 \leqslant r < \infty) \tag{1.2}$$

$$\sigma_{rz}(r, z_0) = \sigma_{rz}(r, z_{N+1}) = 0, \quad \sigma_{zz}(r, z_i) = f_{zi}^*(r) - f_{zi}(r) \quad (0 < r < \infty)$$

$$\sigma_{rz}(r, z_i) = f_{ri}^*(r) - f_{ri}(r) \qquad (0 \leqslant r < a_i)$$

where $z = z_0, z = z_{N+1}$ are the layer boundaries, and $z = z_i$ $(i = 1, \ldots, N)$ are planes containing the middle surfaces of the inclusions S_i, $z_{i+1} > z_i$.

2. We represent the tensor of the state of stress in the layer in the form /5/

$$\sigma = \sum_{i=0}^{N+1} \sigma^i \tag{2.1}$$

Here σ^i $(i = 1, \ldots, N)$ is the stress tensor in unbounded space with a plane slit along S_i, σ^0 and σ^{N+1} are the stress tensors in half-spaces with the boundaries $z = z_0$ and $z = z_{N+1}$, respectively.

The boundary conditions for such auxiliary problems will be the following:

$$\sigma_{zz}^0(r, z_0) = -\sum_{j=1}^{N+1} \sigma_{zz}^j(r, z_0), \quad \sigma_{rz}^0(r, z_0) = -\sum_{j=1}^{N+1} \sigma_{rz}^j(r, z_0) \tag{2.2}$$

$$\sigma_{zz}^{N+1}(r, z_{N+1}) = -\sum_{j=0}^{N} \sigma_{zz}^j(r, z_{N+1}), \quad \sigma_{rz}^{N+1}(r, z_{N+1}) = -\sum_{j=0}^{N} \sigma_{rz}^j(r, z_{N+1})$$

$$(0 \leqslant r < \infty)$$

$$\sigma_{zz}^i(r, z_i) = f_{zi}^*(r) - f_{zi}(r) - \sum_{j=0(\neq i)}^{N+1} \sigma_{zz}^j(r, z_i) \tag{2.3}$$

$$\sigma_{rz}^i(r, z_i) = f_{ri}^*(r) - f_{ri}(r) - \sum_{j=0(\neq i)}^{N+1} \sigma_{rz}^j(r, z_i)$$

$$(0 \leqslant r < a_i), \quad i = 1, \ldots, N$$

As is known, the axisymmetric state of stress in a body can be represented in terms of two harmonic functions Φ_1, Φ_2. The stress tensor and displacement vector components for each state of stress σ^i are expressed in the form

$$2\mu u_z^i = (z - z_i) \frac{\partial^2 (\Phi_1^i + \Phi_2^i)}{\partial z^2} - \frac{\partial \Phi_1^i}{\partial z} - (1 - 2\nu) \frac{\partial (\Phi_1^i + \Phi_2^i)}{\partial z} \tag{2.4}$$

$$2\mu u_r^i = (z - z_i) \frac{\partial^2 (\Phi_1^i + \Phi_2^i)}{\partial r \partial z} + \frac{\partial \Phi_2^i}{\partial r} + (1 - 2\nu) \frac{\partial (\Phi_1^i + \Phi_2^i)}{\partial r}$$

$$\sigma_{zz}^i = (z - z_i) \frac{\partial^3 (\Phi_1^i + \Phi_2^i)}{\partial z^3} - \frac{\partial^2 \Phi_1^i}{\partial z^2}$$

$$\sigma_{rz}^i = (z - z_i) \frac{\partial^3 (\Phi_1^i + \Phi_2^i)}{\partial z^2 \partial r} + \frac{\partial^2 \Phi_2^i}{\partial r \partial z}$$

where μ, ν are the shear modulus and Poisson's ratio of the host material.

We will represent the harmonic functions Φ_1^i, Φ_2^i in the form of Hankel integral expansions

$$\Phi_k^i(r, z) = -\int_0^\infty \xi^{-1} A_k^i(\xi) \exp(-|z - z_i| \xi) J_0(\xi r)\, d\xi, \quad k = 1, 2 \tag{2.5}$$

where A_k^i are unknown functions, and J_0 is a Bessel function of the first kind.

First we consider the boundary value problem (2.2). Substituting the expression for the stress tensor components in the form of the functions Φ_k^i into conditions (2.2), taking account of the integral representations (2.5), and applying the Hankel transformation, we obtain the relations

$$A_1^0 + B_0^{N+1} = -\sum_{j=1}^{N} B_0^j, \quad A_2^0 + C_0^{N+1} = -\sum_{j=1}^{N} C_0^j \tag{2.6}$$

$$A_1^{N+1} - B_{N+1}^0 = -\sum_{j=1}^{N} B_{N+1}^j, \quad A_2^{N+1} - C_{N+1}^0 = -\sum_{j=1}^{N} C_{N+1}^j$$

$$B_k^n = (|z_n - z_m| \xi (A_1^n + A_2^n) + A_1^n) \exp(-|z_n - z_m| \xi)$$

$$C_k^n = (|z_n - z_m| \xi (A_1^n + A_2^n) - A_2^n) \exp(-|z_n - z_m| \xi)$$

$$n, m = 0, 1, \ldots, N + 1$$

We turn to problems for spaces with cracks S_i defined by the boundary conditions (2.3). Considering the case of no tangential and normal stresses in the planes $z_i = 0$ separately, we arrive at the following system of dual integral equations:

$$\int_0^\infty A_1^i(\xi) J_0(\xi r)\, dr = 0 \tag{2.7}$$

$$\int_0^\infty A_2^i(\xi) J_1(\xi r)\, d\xi = 0 \quad (a_i < r < \infty)$$

$$\int_0^\infty \xi A_1^i(\xi) J_0(\xi r)\, d\xi = Q_1^i(r) - \frac{2(1 - \nu^2)\varepsilon_i}{h_i(r)} \int_0^\infty A_1^i(\xi) J_0(\xi r)\, d\xi - \sum_{j=0(\neq i)}^{N+1} \int_0^\infty \xi B_i^j(\xi) J_0(\xi r)\, d\xi$$

$$\int_0^\infty \xi A_2^i(\xi) J_1(\xi r)\,d\xi = Q_2^i(r) - \frac{(1-\nu^2)\varepsilon_i}{(1+\nu_i)h_i(r)} \int_0^\infty A_2^i(\xi) J_1(\xi r)\,d\xi +$$
$$+\sum_{j=0(j\neq i)}^{N+1} \operatorname{sgn}(z_i - z_j) \int_0^\infty \xi C_i^j(\xi) J_1(\xi r)\,d\xi \quad (0 \leqslant r < a_i); \quad i = 1, \ldots, N$$
$$Q_1^i = [u_{zi}^0]\frac{E_i}{2h_i} - f_{zi}, \quad Q_2^i = -[u_{ri}^0]\frac{\mu_i}{2h_i} + f_{ri}, \quad \varepsilon_i = \frac{E_i}{E}$$

where J_1 is a Bessel function of the first kind, and u_{zi}^0 and u_{ri}^0 are components of the vector $\vec{u}_i^0$.

We select the solution of (2.7) in the following form

$$A_1^i(\xi) = \int_0^{a_i} \varphi_i(t)\sin\xi t\,dt, \quad A_2^i(\xi) = \int_0^{a_i} \psi_i(t)\cos\xi t\,dt \tag{2.8}$$

Here $\varphi_i(t)$, $\psi_i(t)$ are functions that are continuous together with their first derivatives in the intervals $[0, a_i]$, and in addition $\varphi_i(0) = 0$. We see by substituting (2.8) into the first three equations in (2.7) that the first equation is satisfied identically for any functions $\varphi_i(t)$ of the class under consideration, and the second is satisfied under the condition

$$\int_0^{a_i} \psi_i(t)\,dt = 0 \tag{2.9}$$

while the third results in the integral equation

$$\int_0^r \frac{\varphi_i'(t)\,dt}{\sqrt{r^2 - t^2}} = Q_1^i(r) - \frac{2(1-\nu^2)\varepsilon_i}{h_i(r)} \int_0^{a_i} \frac{\varphi_i(t)\,dt}{\sqrt{t^2 - r^2}} - \tag{2.10}$$
$$\sum_{j=1(j\neq i)}^{N} \int_0^{a_j} \Big(|z_i - z_j|\Big(\varphi_j(t)\int_0^\infty \xi^2 \sin\xi t J_0(\xi r)\exp(-|z_i - z_j|\xi)\,d\xi +$$
$$\psi_j(t)\int_0^\infty \xi^2 \cos\xi t J_0(\xi r)\exp(-|z_i - z_j|\xi)\,d\xi\Big) +$$
$$\varphi_j(t)\int_0^\infty \xi \sin\xi t J_0(\xi r)\exp(-|z_i - z_j|)\xi)\,d\xi\Big)\,dt - \int_0^\infty (B_i^0(\xi) + B_i^{N+1}(\xi))\,\xi J_0(\xi r)\,d\xi$$

On the basis of the relationships (2.8), the last equation from (2.7) can be reduced by integration with respect to r to the form

$$-\int_0^r \frac{\psi_i(t)\,dt}{\sqrt{r^2 - t^2}} = C_i + \int_0^r Q_2^i(r)\,dr - \frac{(1-\nu^2)\varepsilon_i}{(1+\nu_i)} \int_0^{a_i} \psi_i(t)\,dt \int_0^r T_i(r,t)\,dr - \sum_{j=0(j\neq i)}^{N+1} \operatorname{sgn}(z_i - z_j) \times \tag{2.11}$$
$$\int_0^{a_j} \Big(|z_i - z_j|\varphi_j(t)\int_0^\infty \xi \sin\xi t J_0(\xi r)\exp(-|z_i - z_j|\xi\Big)\,d\xi +$$
$$\psi_j(t)\int_0^\infty (|z_i - z_j|\xi - 1)\cos\xi t J_0(\xi r)\exp(-|z_i - z_j|\xi)\,d\xi)\,dt -$$
$$\int_0^\infty (C_i^0(\xi) - C_i^{N+1}(\xi))\,J_0(\xi r)\,d\xi, \quad T_i = \frac{1}{h_i(r)}\int_0^\infty \cos\xi t J_1(\xi r)\,d\xi$$

On the basis of the known solution of the Abel equation, the relationships (2.10) and (2.11) are reduced to the following integral equations of the second kind:

$$\varphi_i(t) + \frac{4(1-\nu^2)\varepsilon_i}{\pi} \int_0^{a_i} \varphi_i(x)\,dx \int_0^\xi \frac{r\,dr}{\sqrt{(t^2 - r^2)(x^2 - r^2)}\,h_i(r)} + \tag{2.12}$$
$$+\frac{2}{\pi}\sum_{j=1(j\neq i)}^{N} \int_0^{a_j} \Big(\varphi_j(x)\int_0^\infty \big(1 + \xi|z_i - z_j|\big)\sin\xi x \sin\xi t \times \exp(-|z_i - z_j|\xi)\,d\xi +$$
$$\psi_j(x)\int_0^\infty \xi \cos\xi x \sin\xi t \exp(-|z_i - z_j|\xi)\,d\xi\Big)\,dx +$$
$$\frac{2}{\pi}\int_0^\infty (B_i^0(\xi) + B_i^{N+1}(\xi))\sin\xi t\,d\xi = \frac{2}{\pi}\int_0^t \frac{rQ_1^i(r)\,dr}{\sqrt{t^2 - r^2}}$$
$$\psi_i(t) - \frac{2(1-\nu^2)\varepsilon_i t}{(1+\nu_i)\pi} \int_0^{a_i} \psi_i(x)\,dx \int_0^t \frac{T_i(x,r)\,dr}{\sqrt{t^2 - r^2}} -$$

$$\frac{2}{\pi}\sum_{j=1(\neq i)}^{N}\operatorname{sgn}(z_i-z_j)\int_0^{a_j}\Big(\varphi_j(x)\,|z_i-z_j|\int_0^\infty \xi\sin\xi x\cos\xi t\,\times$$
$$\exp(-|z_i-z_j|\,\xi)\,d\xi+\psi_j(x)\int_0^\infty(|z_i-z_j|\,\xi-1)\cos\xi x\cos\xi t\,\times$$
$$\exp(-|z_i-z_j|\,\xi)\,d\xi\Big)\,dx-\frac{2}{\pi}\int_0^\infty(C_i^0(\xi)-C_i^{N+1}(\xi))\cos\xi t\,d\xi=$$
$$-\frac{2}{\pi}C_i-\frac{2}{\pi}t\int_0^t\frac{Q_2^i(r)\,dr}{\sqrt{t^2-r^2}},\quad \zeta=\begin{cases}x, & x<t\\ t, & x>t\end{cases}\quad (i=1,\ldots,N)$$

Having determined the constants C_i by integrating the last equation with respect to t between 0 and a_i, taking condition (2.9) into account, we reduce system (2.12) to the following form

$$\varphi_i(t)+\frac{4(1-\nu^2)\,\varepsilon_i}{\pi}\int_0^{a_i}\varphi_i(x)\,dx\int_0^{\zeta}\frac{r\,dr}{\sqrt{(t^2-r^2)(x^2-r^2)}\,h_i(r)}+ \tag{2.13}$$
$$\frac{2}{\pi}\sum_{j=1(\neq i)}^{N}\int_0^{a_j}(\varphi_j(x)\,K_{1,j}(x,t)+\psi_j(x)\,K_{2,j}(x,t))\,dx+$$
$$\frac{2}{\pi}\int_0^\infty(B_i^0(\xi)+B_i^{N+1}(\xi))\sin\xi t\,d\xi=\frac{2}{\pi}\int_0^t\frac{rQ_1^i(r)\,dr}{\sqrt{t^2-r^2}}$$
$$\psi_i(t)-\frac{2(1-\nu^2)\,\varepsilon_i}{(1+\nu_i)\,\pi}\int_0^{a_i}\psi_i(x)\Big(t\int_0^t\frac{T_i(x,r)\,dr}{\sqrt{t^2-r^2}}-\frac{1}{a_i}\int_0^{a_i}t\,dt\int_0^t\frac{T_i(x,r)\,dr}{\sqrt{t^2-r^2}}\Big)\,dx-$$
$$\frac{2}{\pi}\sum_{j=1(\neq i)}^{N}\operatorname{sgn}(z_i-z_j)\int_0^{a_j}(\varphi_j(x)\,K_{3,j}(x,t)+\psi_j(x)\,K_{4,j}(x,t))\,dx-$$
$$\frac{2}{\pi}\int_0^\infty(C_i^0(\xi)-C_i^{N+1}(\xi))\Big(\cos\xi t-\frac{\sin\xi a_i}{\xi a_i}\Big)\,d\xi=-\frac{2}{\pi}t\int_0^t\frac{Q_2^i(r)\,dr}{\sqrt{t^2-r^2}}+\frac{2}{\pi a_i}\int_0^{a_i}t\,dt\int_0^t\frac{Q_2^i(r)\,dr}{\sqrt{t^2-r^2}}$$
$$K_{1,j}(x,t)=\frac{1}{2}|z_j-z_i|\,(G(x-t)-G(x+t)+D(x-t)-D(x+t))$$
$$K_{2,j}(x,t)=(z_j-z_i)^2\,((x+t)\,D^2(x+t)+(t-x)\,D^2(t-x))$$
$$K_{3,j}(x,t)=(z_j-z_i)^2\,((x+t)\,D^2(x+t)+(x-t)\,D^2(x-t))-{}^1/_2\,|z_j-z_i|\,a_i^{-1}\,(D(a_i-x)-D(a_i+x))$$
$$K_{4,j}(x,t)=\frac{1}{2}|z_i-z_j|\,(-G(x+t)-G(x-t)-$$
$$a_i^{-1}((a_i+x)\,D(a_i+x)-(a_i-x)\,D(a_i-x)))-\frac{1}{2}(|z_i-z_j|\,(D(x+t)+D(x-t))-$$
$$a_i^{-1}\Big(\operatorname{arctg}\frac{a_i+x}{|z_i-z_j|}+\operatorname{arctg}\frac{a_i-x}{|z_i-z_j|}\Big)\Big)$$
$$G(y)=\frac{(z_j-z_i)^2-y^2}{((z_j-z_i)^2+y^2)^2},\quad D(y)=\frac{1}{(z_j-z_i)^2+y^2}$$

The unknown functions $A_1^0(\xi), A_2^0(\xi), A_1^{N+1}(\xi), A_2^{N+1}(\xi)$ are eliminated from the integral equations obtained by using the system of algebraic equations (2.6) and expressions (2.8).

Therefore, the problem for a layer with inclusions has been reduced to solving a system of Fredholm integral equations of the second kind (2.13) and the algebraic equations (2.6). If the functions $\varphi_i(t)$ and $\psi_i(t)$ are found from these equations, then the normal and shear stresses in the planes $z=z_i$ outside the slits S_i can be determined from the expressions

$$\sigma_{zz}^i=\int_0^\infty \xi A_1^i(\xi)\,J_0(\xi r)\,d\xi,\quad \sigma_{rz}^i=-\int_0^\infty \xi A_2^i(\xi)\,J_1(\xi r)\,d\xi \tag{2.14}$$

where $A_1^i(\xi), A_2^i(\xi)$ are found from (2.8).

In particular, the asymptotic expressions for the stresses in the neighbourhood of the contours $(r\to a_i)$ will have the form

$$\sigma_{zz}^i=-\frac{\varphi_i(a_i)}{\sqrt{r^2-a_i^2}}+O(1),\quad \sigma_{rz}^i=-\frac{a_i\psi_i(a_i)}{r\sqrt{r^2-a_i^2}}+O(1) \tag{2.15}$$

Here $O(1)$ is a bounded quantity as $r\to a_i$.

Determining the stress intensity factors K_I, K_{II} from the relationships (2.14), and using the results from /3,4/, we find the magnitude of the normal and tangential stress concentrations in the neighbourhood of the inclusions

$$\sigma_{zz}=f_{zi}-\frac{2\varphi_i(a_i)}{\sqrt{a_i\rho_i}},\quad \sigma_{rz}=f_{ri}-\frac{\psi_i(a_i)}{\sqrt{a_i\rho_i}} \tag{2.16}$$

where ρ_i is the radius of curvature of the apex of the i-th inclusion.

As an illustration, we present the value of a numerical analysis of (2.13) for the case of an infinite space with two identical spheroidal inclusions. A field of uniaxial tension in the direction of the axis of the inclusions is given at infinity. The results are represented in Fig. 2 for different values of the parameter $\beta = a/c$ where $\nu = \nu_1 = 0.3$, and $H/a = 2$.

Also presented for comparison are the data from /6/ (the dashed line) obtained by the method of equivalent inclusion. As is seen from the curves, there is good agreement between the results even for fairly thick inclusions and a broad range of variation of the parameter $\varepsilon_1 = E_1/E$.

The results in /7-9/ follow from (2.13) and (2.16) as special cases.

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THE STANDARD EQUATION METHOD IN THE DYNAMICS OF STRUCTURALLY INHOMOGENEOUS ELASTIC MEDIA*

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The development of the standard equation method is examined for studying harmonic wave propagation in stochastically inhomogeneous elastic media. The Helmholtz operator equation describing the propagation of a mean scalar field in a medium is investigated as the standard equation. For an arbitrary correlation function of the elastic coefficients of the medium, the roots of the dispersion equation are found by expanding them in a series in the dispersion parameter, and the eigenvectors of the operator are correspondingly determined approximately. For media of the exponential class, the roots and eigenfunctions of the standard problem are determined exactly. Results obtained in solving the standard problem, are used in investigating wave propagation in elastic media; the roots and eigenvectors are found in the form of a series expansion in the dispersion. A relationship is set up between the spectra of the elastic operator and the operator of the standard problem. Formulas are obtained to find the mean elastic fields (including the eigenvectors) in terms of the mean standard functions in the form of scattering series.

The elastic operator in an isotropic homogeneous body has eigenvectors in the form of longitudinal and transverse waves satisfying the Helmholtz equations. The eigenvalues and vectors of an elastic operator are a set of eigenvalues and vectors of the Helmholtz operator /1/. The elasticity equations do not split into Helmholtz equations or scalar equations in the general case in an inhomogeneous medium. This can be done for high

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